Are We Serious About Using TLA+ For Statistical Properties?

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Should I put a Mastodon handle here, or BlueSky or what?

Just send me a damn email: jesse@mongodb.com

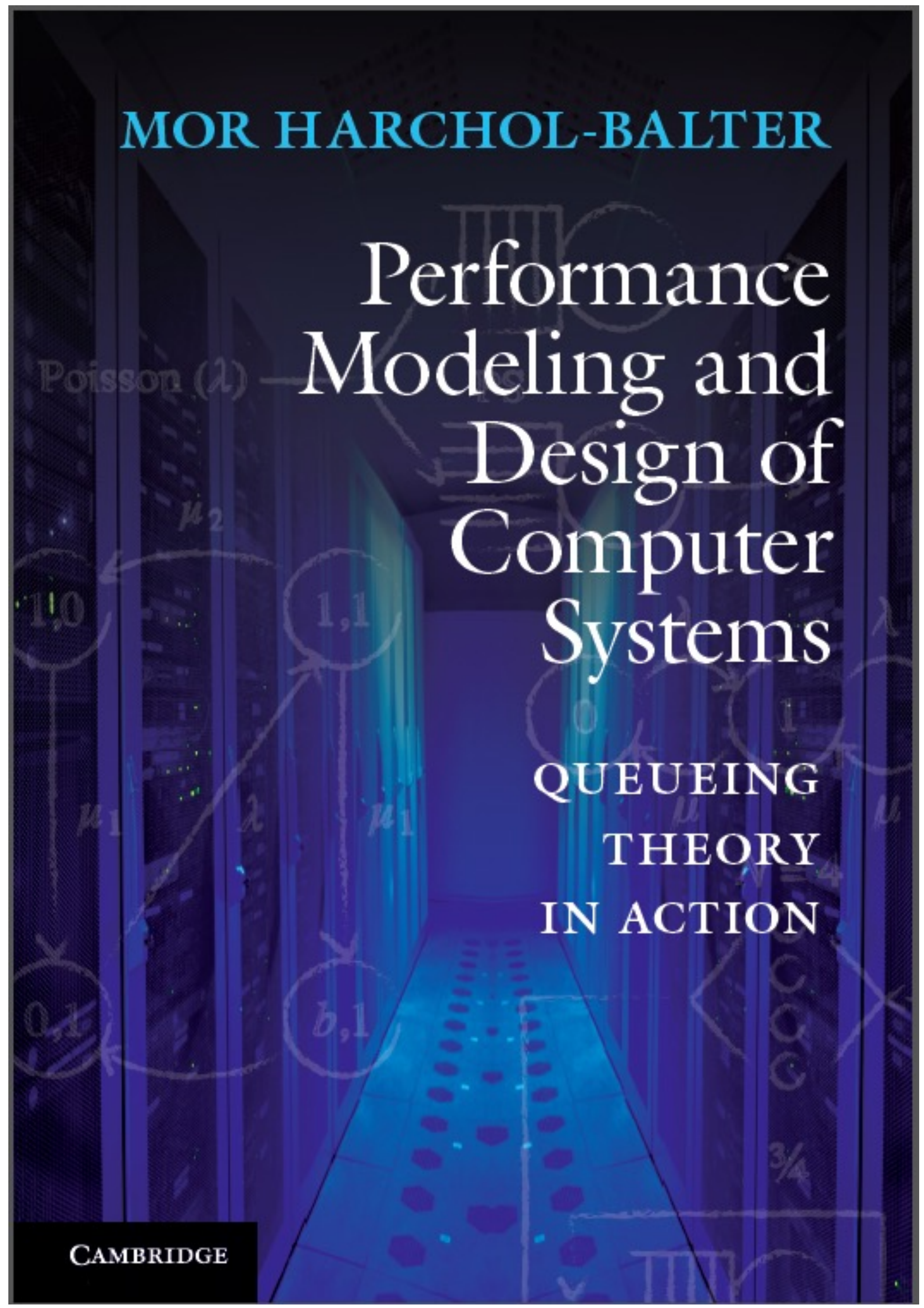
"Formal Methods Only Solve Half My Problems"

Marc Brooker, 2022:

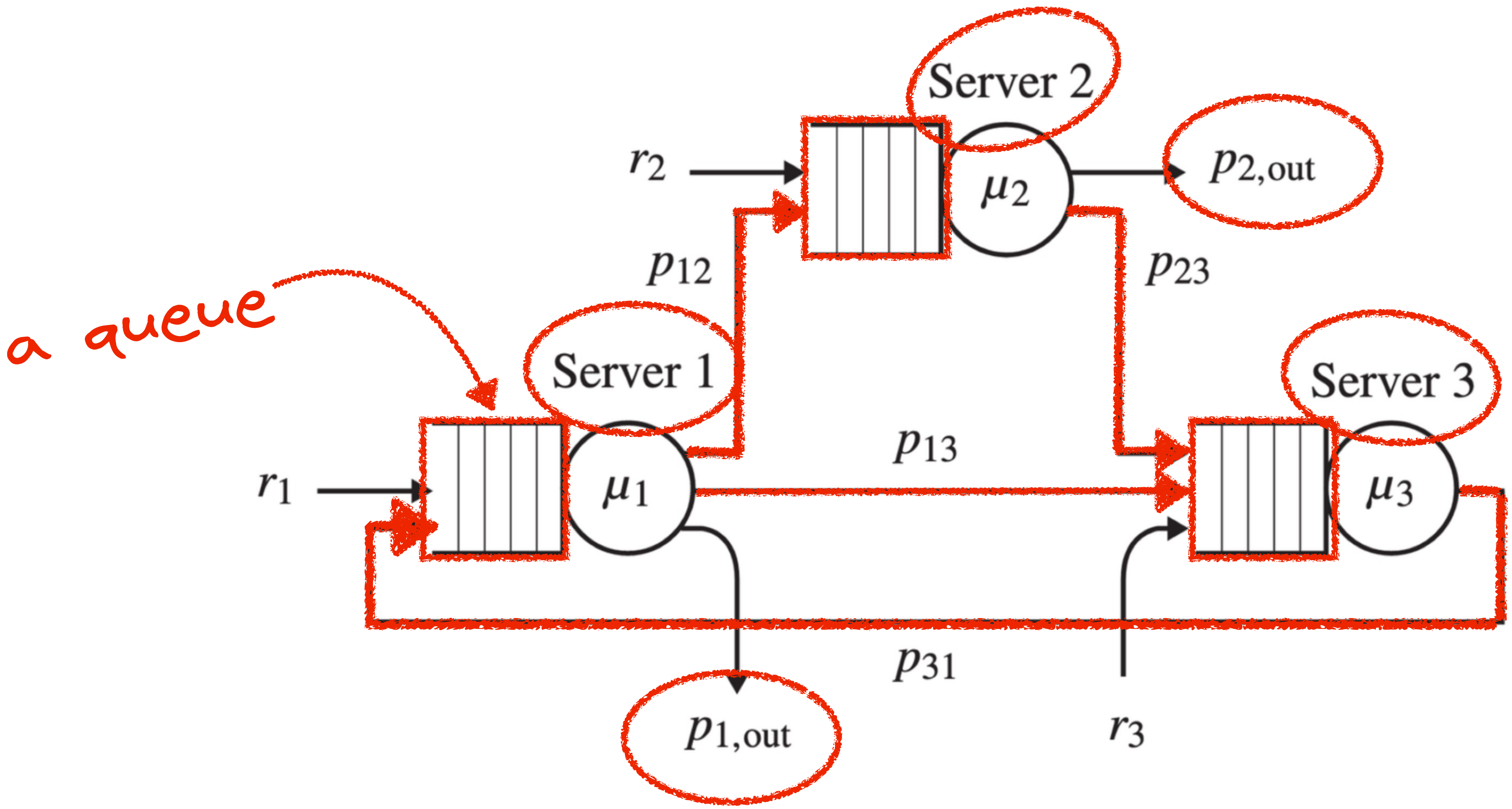
TLA+ can check correctness (safety and liveness), but not performance characteristics.

"What I want is tools that do both: tools that allow development of formal models ... and then allow us to ask those models questions about design performance."

Learn Queueing Theory?



Learn Queueing Theory?



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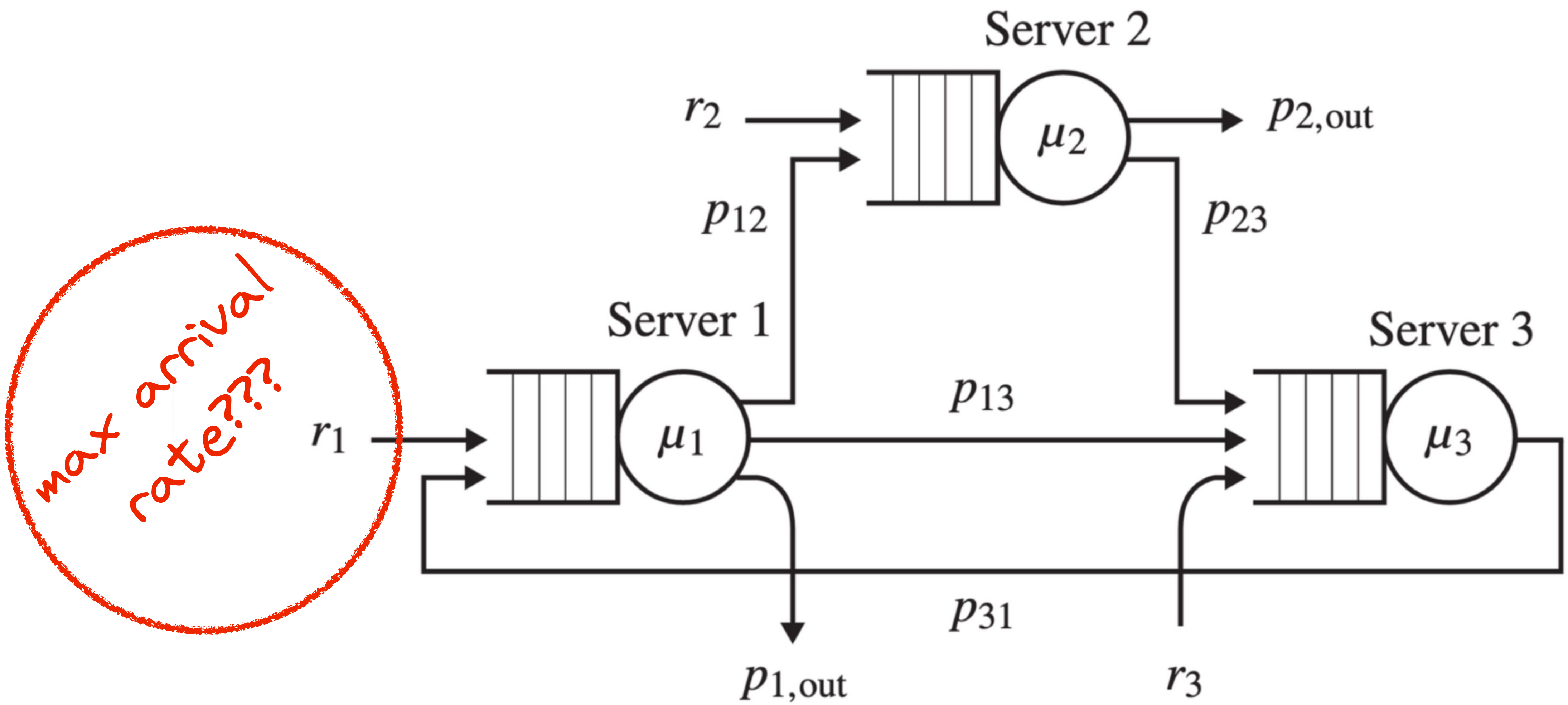
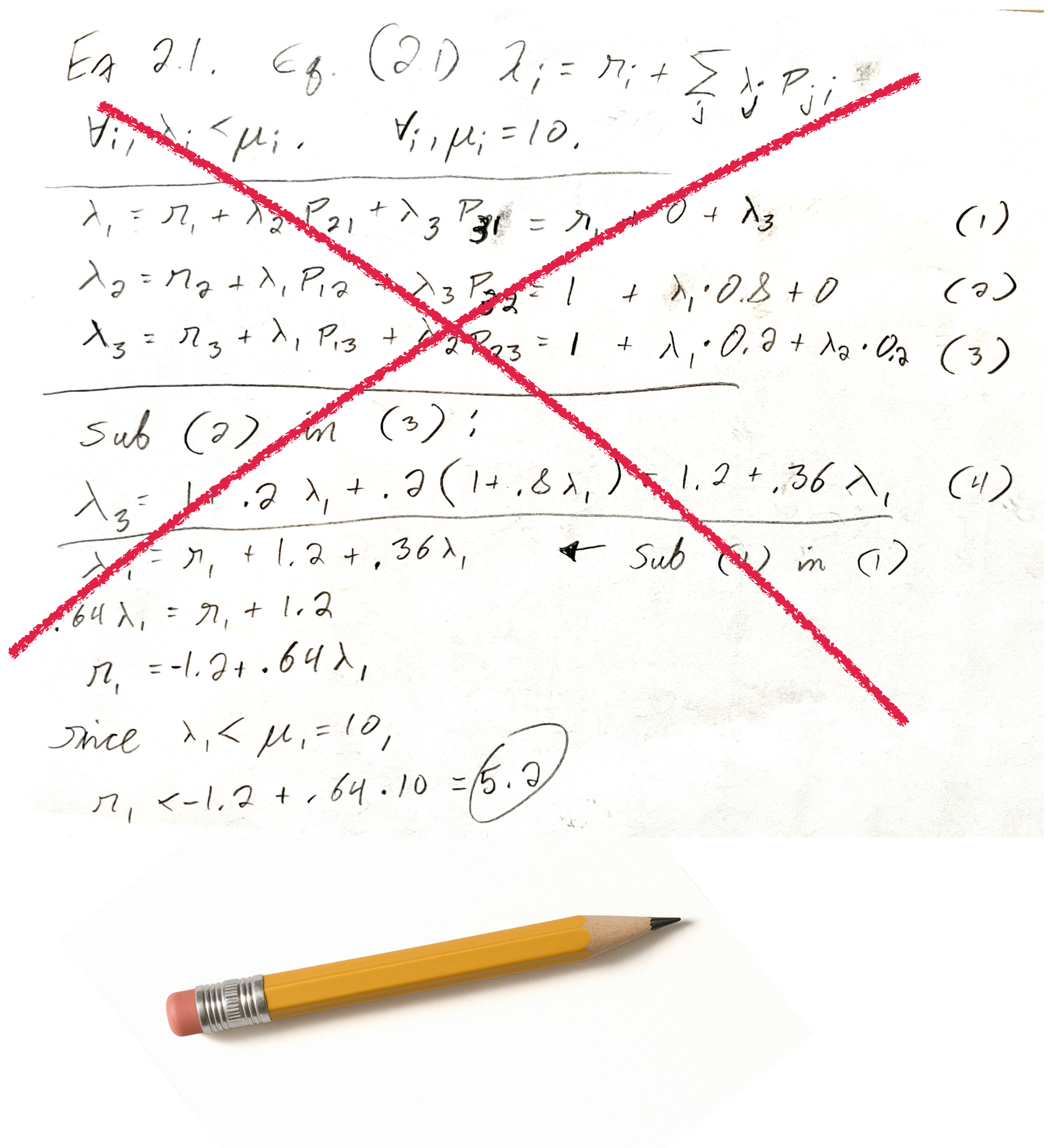
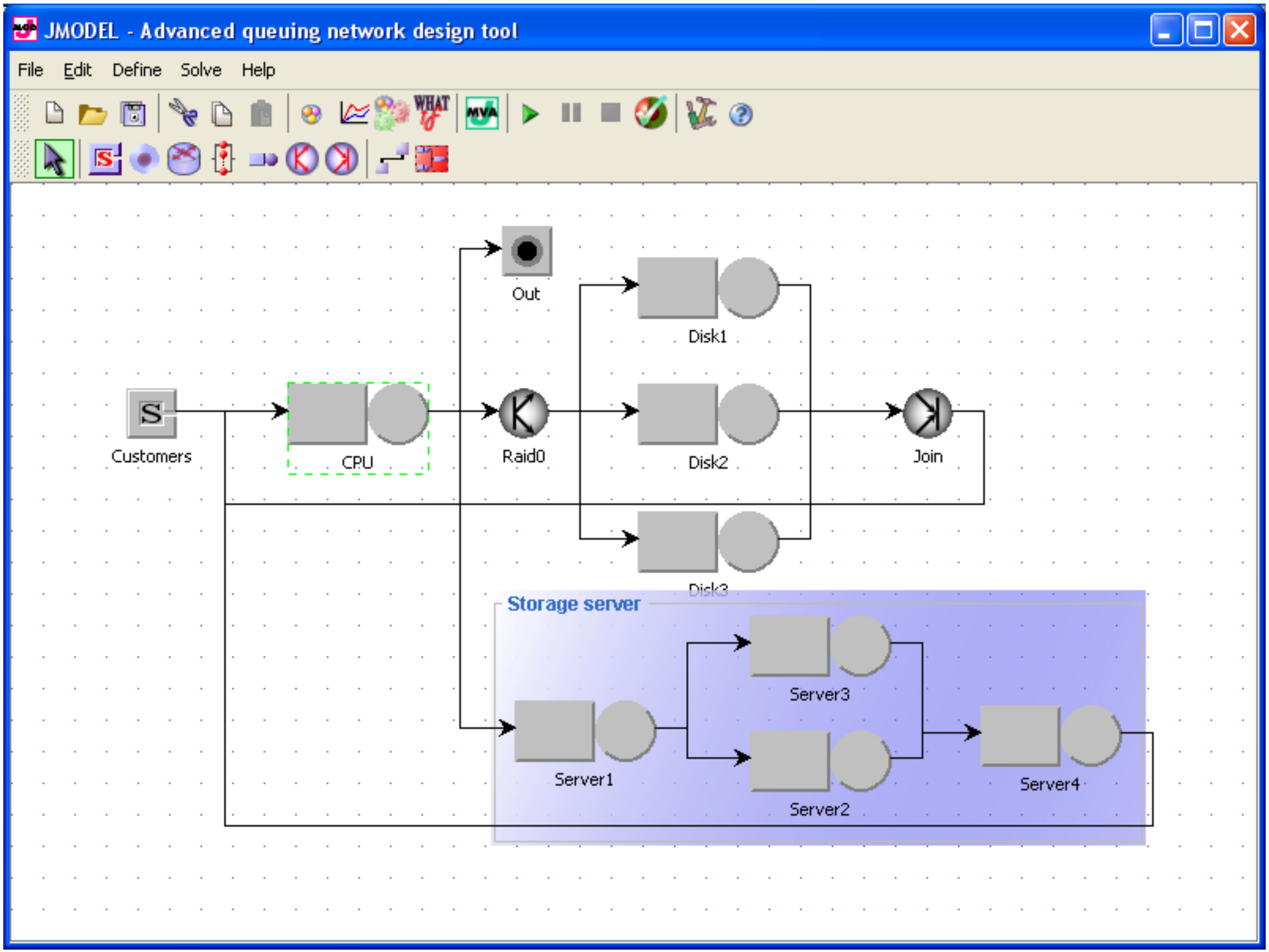


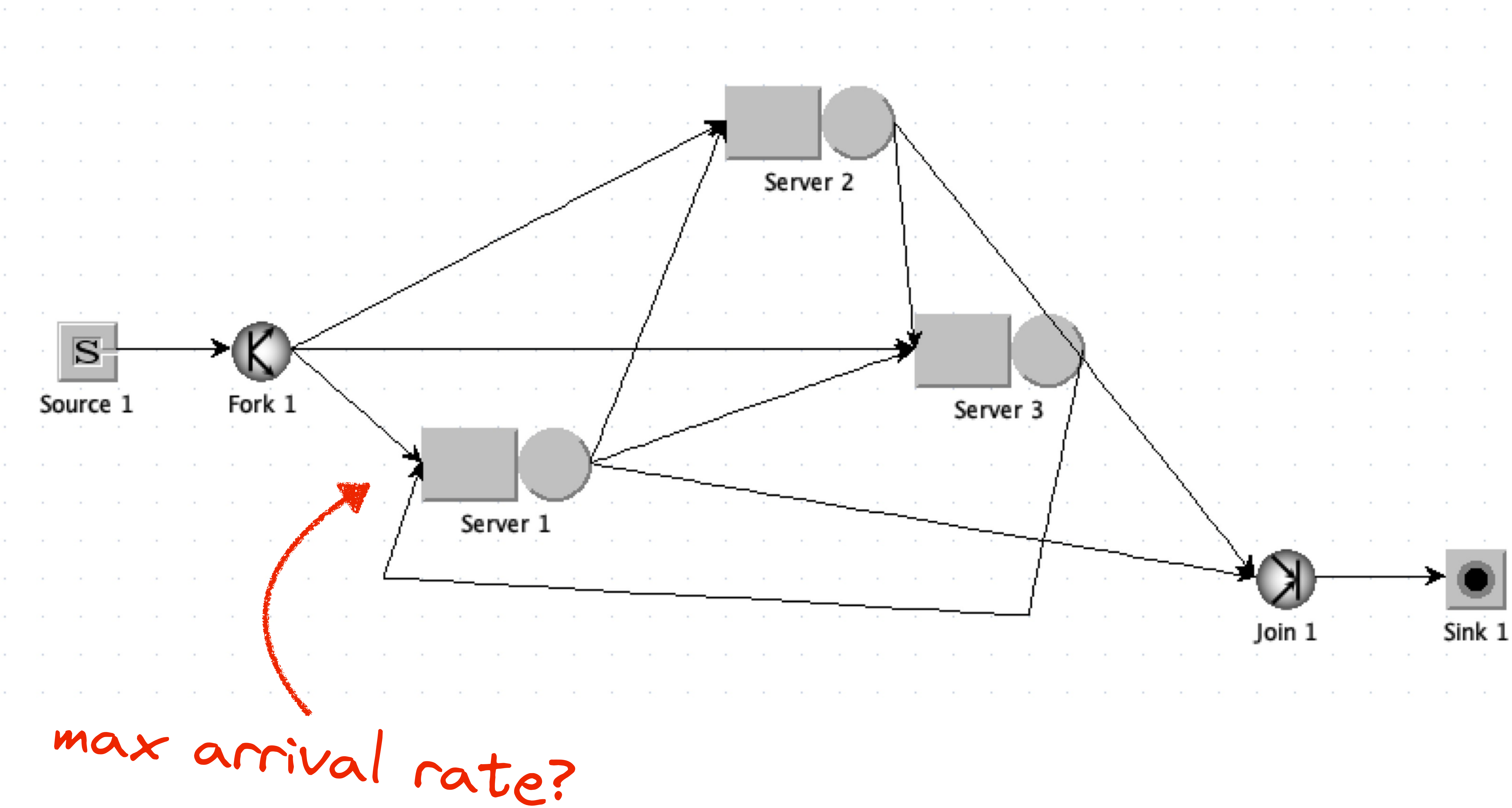
Learn Queueing Theory?



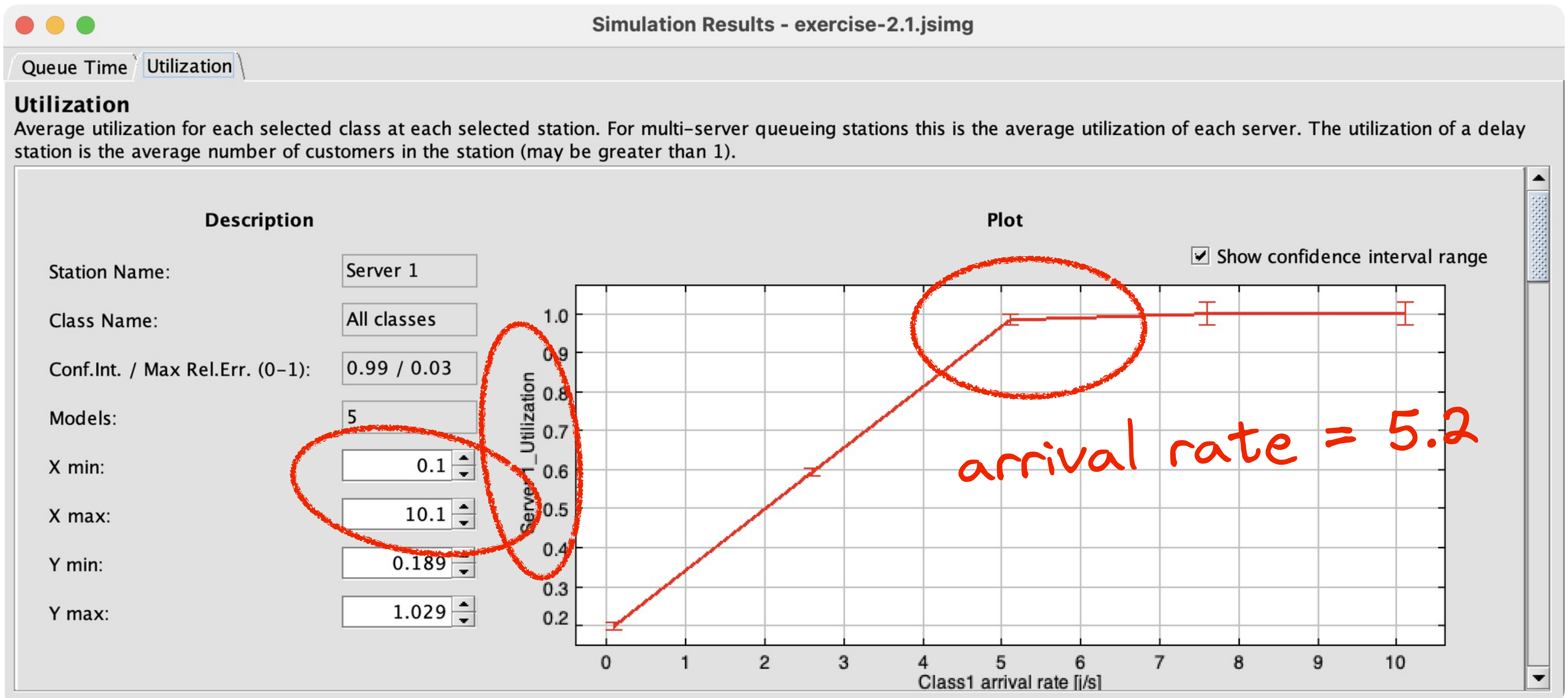
Java Modelling Tools



Java Modelling Tools



Java Modelling Tools



Learn Queueing Theory?

- Queueing theory has super-useful concepts: arrival rate, service rate, utilization, ergodicity, Little's Law, service discipline, open vs. closed loop, and many more.
- Queueing theory math is heinous.
- Don't try to learn the math.
- You can't estimate system performance by solving equations.
- Just run simulations.

Have We Solved All Marc's Problems?

“Formal Methods Only Solve Half My Problems”

“What I want is tools that do both: tools that allow development of formal models … and then allow us to ask those models questions about design performance.”

“Obtaining Statistical Properties via TLC Simulation”

Jack Vanlightly and Markus Kuppe

TLA$^+$ Conference 2022

Updating a statistic

Jack Vanlightly's TLA+ spec of a gossip protocol

```
\* Increment the updates counter by the number of incoming peer states.
TLCSet(updates_ctr_id, TLCGet(updates_ctr_id)
                       + Cardinality(DOMAIN incoming_peer_states))
```

"cost function"

Writing a CSV line

Jack Vanlightly's TLA+ spec of a gossip protocol

```
CSVWrite(
    "%1$s,%2$s,%3$s,%4$s,%5$s,%6$s,%7$s,%8$s,%9$s,%10$s,%11$s,%12$s,%13$s,"
    \o "%14$s,%15$s,%16$s,%17$s,%18$s,%19$s,%20$s,%21$s,%22$s,%23$s,%24$s,%25$s",
    <<behaviour_id,
        r, RoundMessageLoad(r), DirectProbeDeadMessageLoad(r), IndirectProbeDeadMessageLoad(r),
        TLCGet(updates_pr_ctr(r)), TLCGet(eff_updates_pr_ctr(r)), alive_count, suspect_count,
        dead_count, alive_states_count, suspect_states_count, dead_states_count,
        infective_states_count, infectivity, cfg_num_members, cfg_dead_members, cfg_new_members,
        SuspectTimeout, DisseminationLimit, cfg_max_updates, cfg_lose_nth, cfg_peer_group_size,
        cfg_initial_contacts, MaxRound>>,
    RoundStatsCSV)
```

Complaint 1: syntax

Implementing a probability distribution

Jack Vanlightly's TLA+ spec of a gossip protocol

```
\*  'probabilistic' is a random chance of losing the message
\*  'exhaustive' is for model checking where both options are explored
GetDeliveredCount() ==
  CASE MessageLossMode = "probabilistic" ->
      IF RandomElement(1..cfg_lose_nth) = cfg_lose_nth THEN {0} ELSE {1}
    [] MessageLossMode = "exhaustive" -> {0,1}

SendMessage(msg) ==
  \E delivered_count \in GetDeliveredCount() :
    \* ... send the message if delivered_count is 1 ...
```

Complaint 2: randomization is incompatible with model-checking*

*correction: Markus says this is fixed

Complaint 3: randomization is very limited

Complaint 3: randomization is very limited

```
\* In your dreams
TLCSet(cost, TLCGet(cost) + 1)
TLCSet(cost, TLCGet(cost) + 2.5)
TLCSet(cost, TLCGet(cost) + Exponential(3))
```

Complaint 4:

no floats

no probability distributions besides "uniform"

Are We Serious About Statistical Properties?

State of the Art

1. Java Modelling Tools
2. PRISM
3. Runway
4. FizzBee

State of the Art #1 of 4:
Java Modelling Tools

- Comes with an extra L, straight from London, tariff-free.
- Made for statistical modeling and answering performance questions.
- Point-and-click interface — is this a pro or a con? 🤔
- Lots of probability distributions.
- Cost functions.
- Use real-world data sets as inputs!

State of the Art #2 of 4: PRISM

Probabilistic Model Checker

```
[my_action] x=0 -> 0.8:(x'=1) + 0.2:(x'=2);
```

probabilities or rates, for discrete-time or continuous-time models

PRISM
Cost Functions

A “cost” is any measurement of performance. PRISM calls them “rewards”.

```
rewards
    x=0 : 100;
    x>0 & x<10 : 2*x;
endrewards
```

Express good rewards like revenue, or bad costs like latency.

PRISM
Property Expressions

```
P<0.1 [ F<=100 num_errors > 5 ]
```

"the probability that more than 5 errors occur within the first 100 time units is less than 0.1"

```
P=? [ !proc2_terminate U proc1_terminate ]
```

"the probability that process 1 terminates before process 2 does"

PRISM

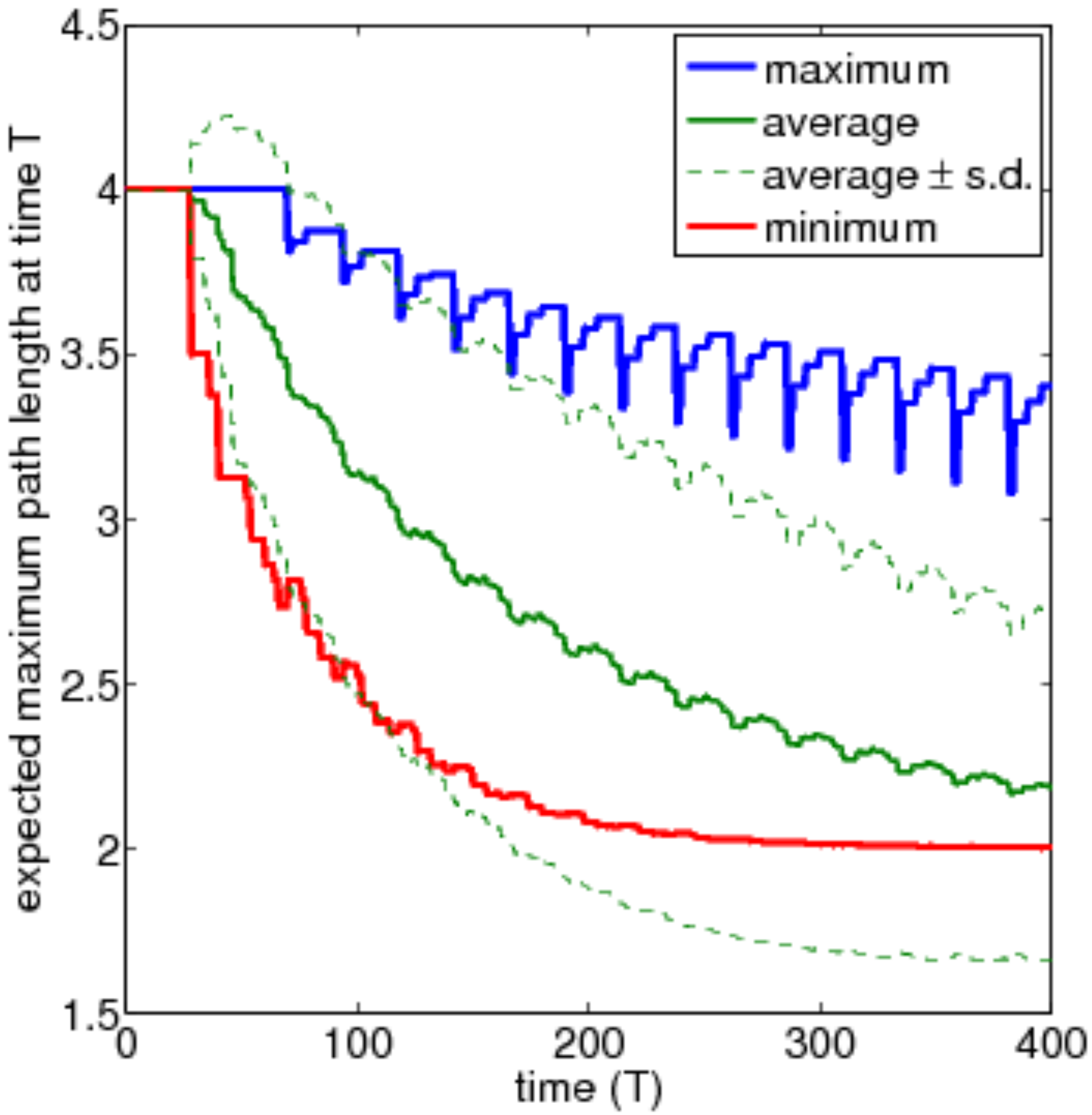
Property Expressions

Safety: long-run probability something bad happens is 0.

Liveness: long-run probability something good happens is 1.

Performance: p95 latency is less than x.

PRISM model of a gossip protocol



PRISM model of a gossip protocol

```
// initial view of node 1 (can see 2 one hop away)
const int iv1_1_a = 2;
const int iv1_2_a = 0;
const int iv1_1_h = 1;
const int iv1_2_h = 4;

// initial view of node 2 (empty)
const int iv2_1_a = 0;
const int iv2_2_a = 0;
const int iv2_1_h = 4;
const int iv2_2_h = 4;

// initial view of node 3 (can see 2 one hop away)
const int iv3_1_a = 2;
const int iv3_2_a = 0;
const int iv3_1_h = 1;
const int iv3_2_h = 4;

// initial view of node 4 (can see 2 one hop away)
const int iv4_1_a = 2;
const int iv4_2_a = 0;
const int iv4_1_h = 1;
const int iv4_2_h = 4;
```

PRISM model of a gossip protocol

Some of Node 1's code:

```
// send to node 2
[push1_2_0] s1=3 & send1=id2 & i1=0 -> (i1'=i1+1);
[push1_2_1] s1=3 & send1=id2 & i1=1 & v1_1_h<4 -> (s1'=0) & (i1'=0) & (send1'=0);
[push1_2_end] s1=3 & send1=id2 & ((i1=1&v1_1_h=4) | (i1=2&v1_2_h=4)) -> (s1'=0) & (i1'=0) & (send1'=0);
// send to node 3
[push1_3_0] s1=3 & send1=id3 & i1=0 -> (i1'=i1+1);
[push1_3_1] s1=3 & send1=id3 & i1=1 & v1_1_h<4 -> (s1'=0) & (i1'=0) & (send1'=0);
[push1_3_end] s1=3 & send1=id3 & ((i1=1&v1_1_h=4) | (i1=2&v1_2_h=4)) -> (s1'=0) & (i1'=0) & (send1'=0);
// send to node 4
[push1_4_0] s1=3 & send1=id4 & i1=0 -> (i1'=i1+1);
[push1_4_1] s1=3 & send1=id4 & i1=1 & v1_1_h<4 -> (s1'=0) & (i1'=0) & (send1'=0);
[push1_4_end] s1=3 & send1=id4 & ((i1=1&v1_1_h=4) | (i1=2&v1_2_h=4)) -> (s1'=0) & (i1'=0) & (send1'=0);
```

PRISM model of a gossip protocol

Some of Node 2's code:

```
push1_2_0=push2_1_0, push1_2_1=push2_1_1, push1_2_2=push2_1_2, push1_2_3=push2_1_3, push1_2_end=push2_1_end,
push1_3_0=push2_3_0, push1_3_1=push2_3_1, push1_3_2=push2_3_2, push1_3_3=push2_3_3, push1_3_end=push2_3_end,
push1_4_0=push2_4_0, push1_4_1=push2_4_1, push1_4_2=push2_4_2, push1_4_3=push2_4_3, push1_4_end=push2_4_end,
push2_1_0=push1_2_0, push2_1_1=push1_2_1, push2_1_2=push1_2_2, push2_1_3=push1_2_3, push2_1_end=push1_2_end,
push3_1_0=push3_2_0, push3_1_1=push3_2_1, push3_1_2=push3_2_2, push3_1_3=push3_2_3, push3_1_end=push3_2_end,
push4_1_0=push4_2_0, push4_1_1=push4_2_1, push4_1_2=push4_2_2, push4_1_3=push4_2_3, push4_1_end=push4_2_end
```

Start of the Art #3 of 4: Runway

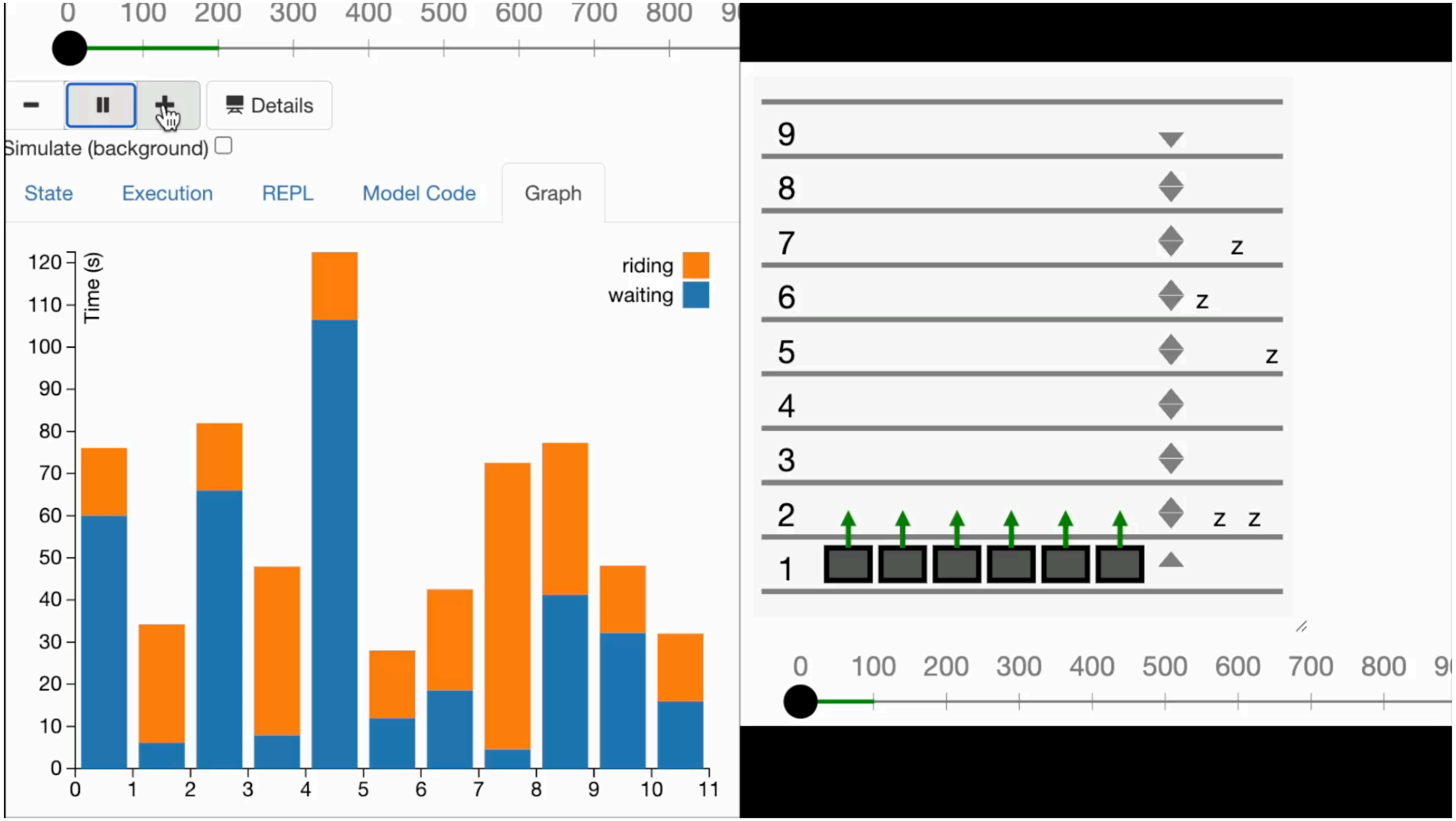
Diego Ongaro

```
function quorum(serverSet: Set<ServerId>[ServerId]) -> Boolean {
  return size(serverSet) * 2 > size(servers);
}

function sendMessage(message: Message) {
  message.sentAt = later(0);
  message.deliverAt = later(urandomRange(10000, 20000));
  push(network, message);
}
```

Runway

Elevator Simulation



State of the Art #4 of 4: FizzBee

Jayaprabhakar "JP" Kadarkarai

cache.fizz

```
atomic action Lookup:
  cached = LookupCache()
  if cached == "hit":
      return cached
  found = LookupDB()
  return found

func LookupCache():
  oneof:
    `hit` return "hit"
    `miss` return “miss"
```

perf_model.yaml

```
configs:
  LookupCache.call:
    counters:
      latency_ms:
        numeric: 10
  LookupCache.hit:
    probability: 0.2
  LookupCache.miss:
    probability: 0.8
```

cost function

probabilities

```
Metrics(mean={'latency_ms': 84.4})
   2: 0.20000000 state: {} / returns: {"Lookup":"\"hit\""}
   4: 0.72000000 state: {} / returns: {"Lookup":"\"found\""}
   5: 0.08000000 state: {} / returns: {"Lookup":"\"notfound\""}
```

perf_model.yaml

```yaml
configs:
  LookupCache.call:
    counters:
      latency_ms:
        distribution: lognorm(s=0.3, loc=2)
  LookupCache.hit:
    probability: 0.2
  LookupCache.miss:
    probability: 0.8
```

Any probability distro supported by SciPy

or bring your own histogram

"Formal Methods Only Solve Half My Problems"

Marc Brooker, 2022:

"What I want is tools that do both: tools that allow development of formal models ... and then allow us to ask those models questions about design performance. Ideally, those tools would allow real-world data on network performance, packet loss, and user workloads to be used, alongside parametric models."

cache.fizz

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atomic action Lookup:
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perf_model.yaml

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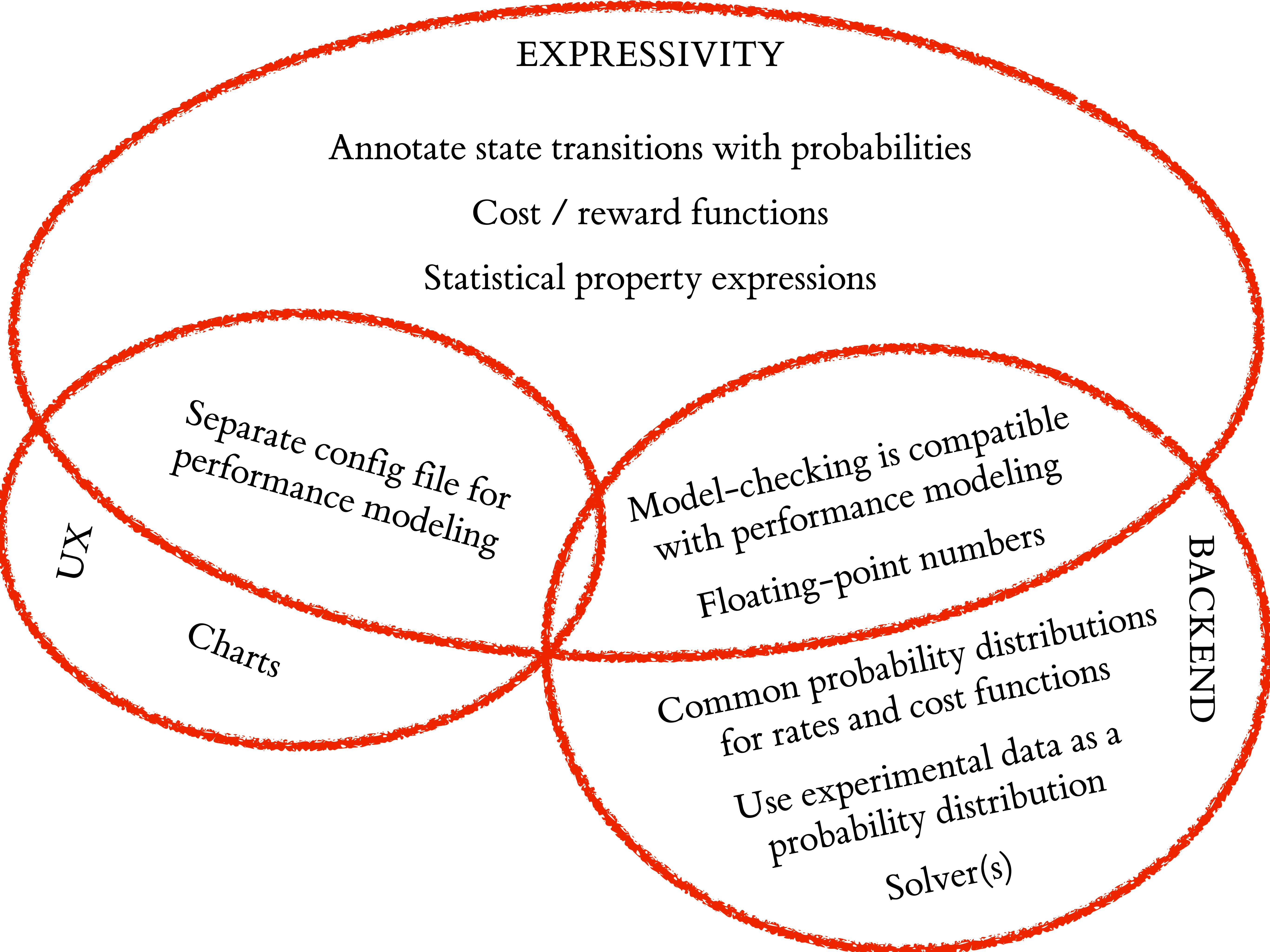
```

cost function

probabilities


EXPRESSIVITY
Annotate state transitions with probabilities
Cost / reward functions
Statistical property expressions
Separate config file for performance modeling
UX
Charts
Model-checking is compatible with performance modeling
Floating-point numbers
BACKEND
Common probability distributions for rates and cost functions
Use experimental data as a probability distribution
Solver(s)

Possible Syntax??

MySpec.tla

```
SendMessage(m) ==
  \E messageIsDropped \in {FALSE, TRUE}:
    ...
```

Possible Syntax??

MySpec.tla

```
SendMessage(m) ==
  \E messageIsDropped \in MessageLossProbability(FALSE, TRUE):
    ...
```

nondeterministically false / true
or a label for a
probability distribution

Possible Syntax??

MySpec.tla

```
SendMessage(m) ==
  \E messageIsDropped \in MessageLossProbability(FALSE, TRUE):
    ...
```

MySpec.cfg

```
DISTRIBUTION
  MessageLossProbability = BooleanChoice(0.23)
```

Possible Syntax??

MySpec.tla

```
SendMessage(m) ==
  \E messageIsDropped \in MessageLossProbability(FALSE, TRUE):
    ...
```

MySpec.cfg

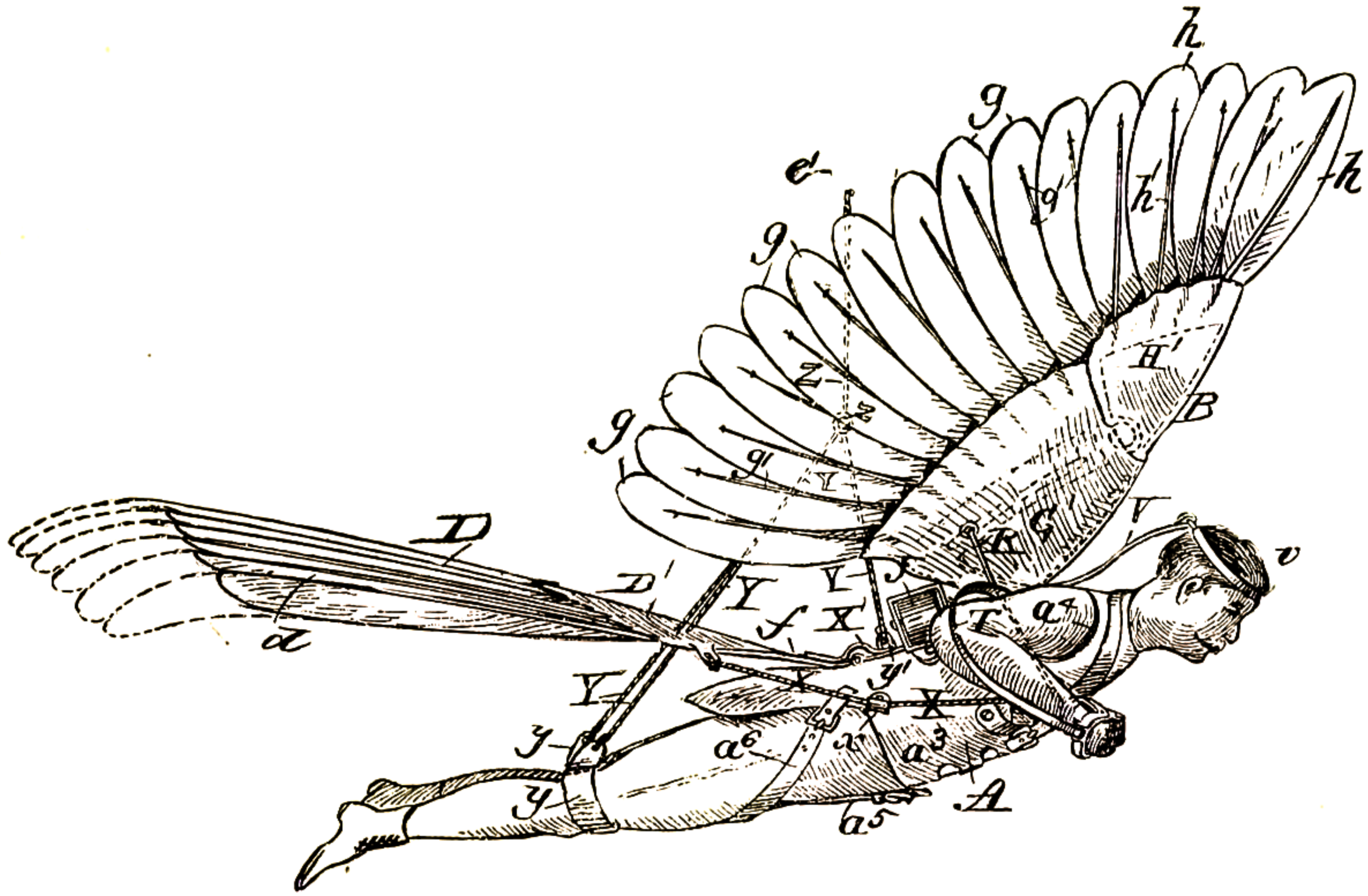
```
DISTRIBUTION
  MessageLossProbability = BooleanChoice(0.23)

COST
  SendMessage = Exponential(3.17)
```

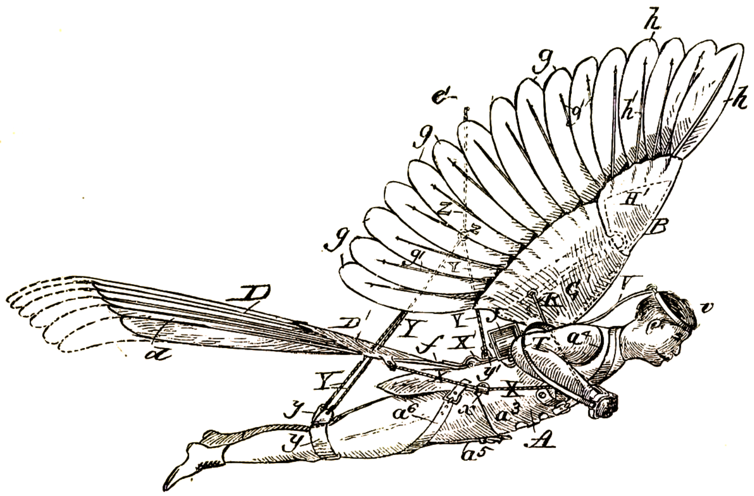
TLA+ with Probabilistic Solvers

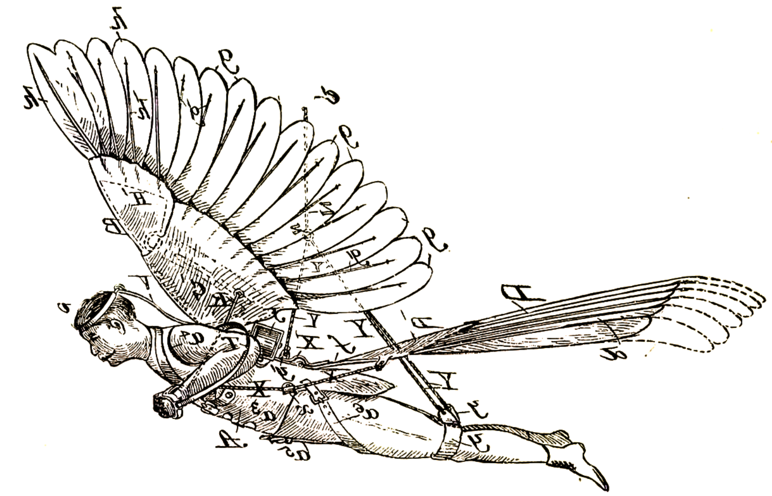
In order of ambitiousness....

- Just use `-generate`, generate thousands of behaviors, average the stats.
- Use `-generate`, run until stats stabilize within some precision, perhaps prune branches of the state graph as they stabilize.
- Use PRISM's solvers (by translating the state graph to PRISM?).
- Write a solver or solvers from scratch: translate the state graph to a Markov chain and find its steady-state probability distribution.



TLA⁺ with Performance Modeling





One model could:

- Express the algorithm.
- Check correctness.
- Evaluate performance.
- Simulate "what-if" experiments using real-world inputs.
- Confidently explore optimizations.

Acks

- Andrew Helwer
- Jayaprabhakar Kadarkarai
- Murat Demirbas
- Will Schultz

Questions

1. What syntax should TLA+ use for annotating state transitions with probabilities?

2. What syntax for cost functions?

3. How do we separate performance-modeling config from the spec and model-checking config?

4. Should TLC do the probabilistic checking, or another tool?

5. Could the TLA+ Foundation get new funding for this work?

6. Is any of this a good idea or should TLA+ stick to correctness?